

# The Undulator Radiation Collider: An Energy Efficient Design For A $\sqrt{s} = 10^{15}$ GeV Collider

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## Abstract

We discuss the main factors affecting the design of accelerators aiming to investigate physics at the GUT scale. The most important constraints turn out to be the energy used and the time taken to accumulate sufficient luminosity. We propose a photon collider design, where the photons are generated by undulator radiation from high energy muon beams. This reduces the energy requirements by a factor of  $10^5$  compared to a  $pp$  collider. A further large reduction in energy use can be achieved by using a periodic magnetic field to prevent a cascade of secondary reactions at the collision points. The proposed collider would be powered by a Dyson swarm constructed around the Sun, and efficient use of energy will be important to reduce the time needed to reach the desired number of collisions.

## 1 Introduction

In the distant future we may wish to construct a collider with a centre of mass energy  $\sqrt{s}$  around the scale of grand unified theories (the GUT scale; approximately  $10^{15}$  GeV) in order to investigate physics at these energies directly. This paper discusses the problems affecting the design of such a collider and proposes an undulator radiation collider as an energy efficient solution.

Throughout this paper, we will assume the Standard Model is correct up to the GUT scale; otherwise the design of (and motivation for) a GUT scale collider would depend on the new physics. The remainder of this section justifies the energy and luminosity required for a GUT scale collider. Section 2 explains why a photon collider is a good choice. Section 3 describes a potentially serious problem due to secondary reactions in the interaction region and how this can be mitigated by using a periodic magnetic field. The detectors and accelerators are described in Sections 4 and 5 respectively, and Section 6 describes the Dyson swarm and energy transmission. Finally we sum up in Section 7.

### 1.1 Target energy and luminosity

#### Energy

After the discovery of the Higgs boson, it is not clear at what energy scale new physics might next be found. Naturalness suggests that this scale should be as low as possible, and preferably at the weak scale ( $\sim 100$  GeV); however this is already in tension with current limits and it is possible that naturalness arguments are simply wrong. Both dark matter and dark energy require physics beyond the Standard Model, but this physics might occur at any scale.

An indication for a scale comes from the fact that the three Standard Model couplings become approximately equal in the range  $10^{14}$ – $10^{15}$  GeV. This observation led to the idea of grand unified theories, in which the three Standard Model gauge groups become unified into one around this scale. The simplest non-supersymmetric GUTs have been ruled out, and in this article we are assuming low-energy supersymmetry does not exist. However, more complicated non-supersymmetric GUTs are still possible [1], and in any case the crossing of the couplings does suggest that something interesting may happen at this scale.

Neutrino masses also provide an indication of a scale at which new physics might be expected. If the see-saw mechanism is correct, these would be generated at a scale of around  $10^{14}$  GeV [1]. The see-saw mechanism can only work if neutrinos are Majorana particles, which is currently unknown; however, it is another indication that there may be new physics in the range  $10^{14}$ – $10^{15}$  GeV.

To be specific, in this paper we will adopt a target energy of  $10^{15}$  GeV, at the upper end of the ranges above. However, the design proposed here could easily be adapted to somewhat lower or higher energies.

## Luminosity

In addition to having sufficient energy, a collider must have a large enough luminosity to discover new physics. The required integrated luminosity depends on the cross-section of interest, which is naturally of order

$$\sigma \sim \frac{\alpha^2}{E^2}, \quad (1)$$

where  $\alpha$  is the relevant coupling constant and we assume that the leading order Feynman diagram includes two vertices. Taking, say  $\alpha = 0.1$ , we find  $\sigma = 4 \times 10^{-64} \text{ m}^2 = 4 \times 10^{-36} \text{ barn}$ .

The number of events observed is  $N = \sigma L$ , where  $L$  is the integrated luminosity. Assuming we want to observe at least 10 or so events, this translates to the requirement  $L \gtrsim 2.5 \times 10^{64} \text{ m}^{-2} = 2.5 \times 10^{18} \text{ ab}^{-1}$ . To put this into perspective, the total integrated luminosity at the LHC is expected to be around  $3 \text{ ab}^{-1}$ .

## 2 Choice of particles

We now turn to the choice of particles to collide. All colliders to date have used either electrons or protons (and sometimes their antiparticles). Of these protons are clearly better at high energy since the synchrotron radiation losses are smaller by a factor  $\left(\frac{m_p}{m_e}\right)^4 \approx 10^{13}$ . This is a significant consideration even for a linear collider, since the beams have to be bent in order to be focused at the interaction points. For example, for electrons with energy  $5 \times 10^{14}$  GeV, a beam of width  $1 \mu\text{m}$  would have to be focused over a length of at least  $10^{11} \text{ m}$  to avoid losing most of its energy by synchrotron radiation.

The total cross-section at  $\sqrt{s} = 10^{15}$  GeV for proton-proton collisions is approximately

$1200 \text{ mb}$  [1]. For an integrated luminosity of  $2.5 \times 10^{18} \text{ ab}^{-1}$ , this corresponds to a total of  $3 \times 10^{36}$  collisions over the lifetime of the experiment. The energy required per collision is  $10^{15}$  GeV (assuming 100% efficiency!); therefore the total energy required is approximately  $5 \times 10^{41} \text{ J}$ . Since the luminosity of the Sun is  $3.8 \times 10^{26} \text{ W}$ , the total energy output of the Sun would be required for some 40 million years.

This is a lot of energy, and it would be nice to be able to use a bit less. The energy required is proportional to the total cross-section, so we should aim to use particles with as small a cross-section as possible.

Using electrons is almost certainly impossible due to the huge synchrotron radiation losses. For other charged particles  $X$ , the process  $X^+X^- \rightarrow X^+X^-e^+e^-$  will contribute to the total cross-section. To leading order, the cross-section for this process is [2]

$$\sigma_{X^+X^- \rightarrow X^+X^-e^+e^-} = \frac{28Z^4\alpha^4}{27\pi m_e^2} \left( \ln \left( \frac{2E}{m^2} \right) \right)^3, \quad (2)$$

where  $E$  is the beam energy and  $Z$  is the charge of  $X$ . This decreases only very slowly with the particle mass  $m$ . For muons it is  $500 \text{ mb}$  and for  $\tau$  particles it is still  $400 \text{ mb}$ , so the gain compared to protons is relatively small. Many hadrons are heavier, but these will have large additional contributions to the cross-section from the strong interaction. The  $W$  boson is heavier still, but its lifetime is far too short. So no known charged particle will do.

The alternative is to collide neutral particles, which can have far smaller cross-sections. However, they cannot be accelerated electromagnetically. Instead they must be produced indirectly, or produced at low energy and then accelerated by colliding them with a high energy beam.

Neutrinos have an extremely low cross-section ( $220 \text{ pb}$  for  $\nu\bar{\nu}$  and  $70 \text{ pb}$  for  $\nu\nu$  [3]), and high energy neutrinos can be produced by the decay of high energy muons. However, there is a problem at the energies we are considering – muons are too stable! At an energy of  $5 \times 10^{14} \text{ GeV}$ , muons travel approximately  $3 \times 10^{18} \text{ m}$  (300 light years) before they decay, requiring a collider at least twice that size (one decay length for each arm). Even worse, the angular spread of the neutrino beam, which is of order  $10^{-16}$ , will result in very wide neutrino beams at the interaction points.

Alternatively we might consider, for exam-

ple, the reaction  $\mu^+\mu^- \rightarrow H^0$  to produce high energy Higgs bosons some centimetres from the collision point (Higgs bosons have a range of approximately 20 cm at  $5 \times 10^{14}$  GeV) or  $\mu^+\mu^- \rightarrow \Upsilon(3S)$  to produce high energy bottomonium tens to hundreds of metres from the collision point ( $\Upsilon(3S)$  particles have a range of approximately 500 m at  $5 \times 10^{14}$  GeV). Higgs bosons are weakly-interacting, and bottomonium, although strongly interacting, is spatially much smaller than a proton, so both these options would reduce the total cross-section. With this method the particles should be produced at resonance so that energy is not wasted producing other, unwanted, types of particles. Nevertheless, for Higgs bosons the process will be very inefficient: the cross-section for  $\mu^+\mu^- \rightarrow H^0$  at resonance is 40 pb [4], whereas the cross-section for process (2) is 2 mb at the same centre-of-mass energy, giving a Higgs boson production efficiency of 0.000002%. Similarly,  $\Upsilon(3S)$  production would also have a low efficiency.

Another problem is that there may be interactions between the produced particles and the incoming beams, e.g. the reaction  $H^0\mu \rightarrow \mu W^+W^-$ , which would reduce the intensity of the beam produced. Whether such reactions are a serious problem requires further investigation.

## Photons

The cross-section for  $\gamma\gamma \rightarrow \text{hadrons}$  at  $\sqrt{s} = 10^{15}$  GeV is only about 0.012 mb [1]. The reaction  $\gamma\gamma \rightarrow e^+e^-e^+e^-$  contributes an additional 0.006 mb [5], giving a total of 0.018 mb. This reduces the energy requirements for a photon-photon collider by a factor of 70000 compared to protons and 30000 compared to muons.

$\gamma\gamma$  colliders have been proposed with centre of mass energies in the GeV to TeV range (see e.g. [6, 7]). The high energy photons in these designs would be produced by inverse Compton scattering: a low energy laser beam would be bounced off a high energy electron beam to produce high energy photons. The limit on this process is pair production of electron-positron pairs by interactions between the incoming and outgoing photons; to avoid this, one must have

$$x = 4 \frac{E_e - E_{\gamma, \text{in}}}{m_e^2} < 4.8. \quad (3)$$

The maximum outgoing photon energy is given by  $E_{\gamma, \text{out}} = \frac{x}{x+1} E_e$ , and is 83% of the electron beam energy for  $x = 4.8$ .

To use this method at extremely high energies, it seems all we have to do is to bounce the photons off muon beams rather than electron beams to avoid the issue of synchrotron losses. Unfortunately, this doesn't work. The limit  $x = 4.8$  applies when the masses of the particles being pair-produced and those in the incoming beam are the same. This is the case for photon collider designs using electrons, but not for our situation: the incoming beam contains muons but the limit on  $x$  is given by the lightest charged particles that can be pair-produced, which are electrons. The limiting value of  $x$  in this situation is

$$x = 2 \left( \frac{m_e}{m_\mu} \right)^2 + 2 \sqrt{\left( \frac{m_e}{m_\mu} \right)^2 + \left( \frac{m_e}{m_\mu} \right)^4} \approx 0.0097. \quad (4)$$

Therefore the maximum outgoing photon energy is over 100 times smaller than the muon beam energy. This means an increase in energy requirements by a factor of 100, reducing the gain to a factor of 1000 compared with protons, or 400 compared with muons. Furthermore, synchrotron losses will increase by a factor of  $10^8$ , unless the curvature radii are increased by a factor of 10000 from the already very large radii required at a beam energy of  $5 \times 10^{14}$  GeV, and the accelerating sections will need to be 100 times longer. These disadvantages seem likely to outweigh the benefit of a reduced energy requirement.

Fortunately, there is an alternative mechanism for producing high energy photons: synchrotron radiation. The question is whether two sufficiently intense beams of synchrotron radiation can be collided in a small enough area. To achieve this it is desirable for the photons to be emitted into a narrow cone and for the emission region to be short so that the beam does not diverge very much. The angular width most of the synchrotron radiation is emitted into is  $\theta \sim 1/\gamma$ , which is indeed very small for the energies we are concerned with. To reduce the length of the emission region the magnetic field strength should be large.

For the analysis of the collisions it would also be desirable for the photon spectrum to be narrow, and to reduce the length of the accelerator the photon energy should not be too far below the energy of the charged particle. Both of these properties can also be achieved by using strong magnetic fields. Specifically, the magnetic field strength in the rest frame of the radiating par-

ticle, in units of the critical field strength, is

$$\chi = \frac{\gamma B}{B_0} \quad (5)$$

where  $B_0$  is the QED critical magnetic field strength, which is  $4.4 \times 10^9$  T for electrons and scales as  $m^2$ . For  $\chi < 1$ , the synchrotron radiation has a broad spectrum centred around  $\chi E$ , (where  $E$  is the energy of the charged particle), but for  $\chi \geq 1$ , the spectrum becomes increasingly narrow and peaks just below  $E$  – see, for example, Figure 15.1 in [8]. Therefore passing a beam of charged particles through a very strong magnetic field would produce the desired photon spectrum.

However, there is a serious difficulty with this. If the magnetic field is stronger than the critical field strength for the radiating particles, which must be muons or heavier particles (electrons presumably being ruled out as discussed above), it will be very much above the critical field strength for electrons. Therefore the photons will be able to pair-produce electron-positron pairs. The photon beam will be attenuated, with the attenuation coefficient per unit length given by [9]

$$\alpha = \frac{\alpha_{EM} m_e c}{2\hbar} \frac{B}{B_0} T(\chi), \quad (6)$$

where  $T(\chi) \approx 0.60\chi^{-1/3}$  for large  $\chi$ , and here  $\chi = \frac{0.5E_\gamma}{m_e c^2} \frac{B}{B_0}$  and the critical field strength for electrons should be used. The length scale over which a particle of energy around  $5 \times 10^{14}$  GeV loses most of its energy to synchrotron radiation is rather small even for moderately strong magnetic fields,  $l_{\text{loss}} \approx \frac{400}{B^{2/3}}$  m, where  $B$  is measured in Teslas. Over this length scale the loss due to electron-positron pair production is significant: the photon intensity is attenuated to  $e^{-l_{\text{loss}}\alpha} \approx 30\%$ . Some of the electrons and positrons will then radiate again, so the overall efficiency is slightly better than this, but this is still rather inefficient.

## Undulator design

An alternative way to achieve a narrow spectrum is to use an undulator – a setup in which the magnetic field is periodic with period  $\lambda_u$  and causes the charged particles to have a periodic trajectory, usually sinusoidal. If there are  $N$  periods the bandwidth of the emitted radiation is  $\sim \frac{1}{N}$  centred at a wavelength approximately

given by

$$\lambda \approx \frac{\lambda_u}{2\gamma^2}, \quad (7)$$

where  $\gamma$  is the Lorentz factor of the primary particles. Note that the radiated wavelength is independent of the magnetic field strength; however, the radiated intensity will be very small unless  $B$  is such that the corresponding synchrotron radiation spectral density is appreciable at  $\lambda$ . Thus if we do not want  $E$  to be very much greater than  $E_\gamma$ , we must have  $\chi$  not much smaller than 1.

The best particles to use as the primaries are probably muons. This is because (again, assuming electrons are impractical) all other particles that are sufficiently stable to accelerate are hadrons, for which pion emission [10] would compete with synchrotron radiation.

Ideally, the muon energy  $E$  would be only slightly higher than  $E_\gamma$ . In this case recoil effects will be very significant. These will reduce the coherence of the undulator [11]: after the muon has emitted a photon its velocity will be reduced and the relative phase between the muon and photon will no longer increase by  $2\pi$  over one undulator period. However, this does not matter: most of the initial muon energy will already have been transferred to the first photon emitted, so what happens to the muon after that is not important (and much of its remaining energy can be recovered in the absorbers described in section 4).

A further advantage of an undulator is that it will suppress pair production. This is because the relative phase between the photon and the electron-positron pair will in general not be a multiple of  $2\pi$  per undulator period, so over many periods there will be destructive interference, reducing the amplitude for pair production.

The undulator parameters — the muon energy, magnetic field, number of periods, and shape, which need not be sinusoidal — will determine the photon energy, photon bandwidth, angular width of the photon beam, and the pair production rate. The actual choice of parameters will be a trade-off between these properties and will depend on the details of future technological development. However, a guess at reasonable parameters shows that they do not seem to be too challenging; indeed, some of them are already possible with present-day technology. For example, for a muon energy of  $6 \times 10^{14}$  GeV, the undulator wavelength, given

by (7), is 160 m, and the required magnetic field is 0.02 T. The energy loss distance with this field is  $\approx 7000$  m, which would give  $N \approx 50$  undulator periods.

To achieve a narrow bandwidth the number of periods can be increased by using a design of the type shown in Figure 1, where the magnetic field is only non-zero for a small part of each undulator period. However, if  $l_B$  is too small, pair production will not be suppressed. This is because the phase per undulator period,  $\phi$ , goes as  $\theta^2 \propto l_B^2$  while the number of undulator periods goes as  $N \propto \frac{1}{l_B}$ . Thus for sufficiently small  $l_B$  the total phase over the length of the undulator,  $N\phi$ , will become small and there will be no cancellation. This only occurs for very small  $l_B$ , however. For example, for  $l_B \approx 2$  m we have  $N \approx 1600$  and  $\phi \approx 0.05$  for symmetrical electron-positron pairs<sup>1</sup>, so  $N\phi \gg 2\pi$  and there will be suppression of pair production.

Nevertheless there will still be some pair-production, and we do not want the electrons and positrons to enter the interaction region. The same goes for any remaining muons. These particles can be swept aside by a short section of magnetic field. Since the bending angle is proportional to  $B$  but pair production is pro-

portional to  $B^{2/3}$  for strong fields [8] this field should be as intense as possible to prevent further pair production occurring within it. For example, a field of  $10^4$  T and length 0.01 m would bend the particles through an angle  $\approx 5 \times 10^{-14}$  with pair production losses of less than 2%. The distance between the synchrotron conversion region and the interaction region is limited by the requirement that the photon beam does not become too wide. The angular width of the photon beam is approximately  $1/\gamma\sqrt{N}$ , where  $N$  is the number of undulator periods. For the parameters above, and assuming a maximum permissible spread of  $10^{-10}$  m, the distance to the interaction region can be up to  $3 \times 10^7$  m. The electrons, positrons and muons would thus be deflected by  $\sim 10^{-6}$  m, which should be sufficient.

In conclusion, the most feasible design for a GUT-scale collider is probably a photon-photon collider, with the photons produced by synchrotron radiation from a muon beam of slightly higher energy than the desired photon energy. By using a suitably designed undulator a narrow bandwidth, small angular width, and a low pair production rate can be achieved.

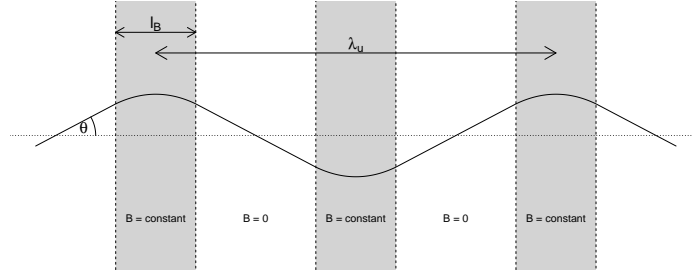


Figure 1: Possible undulator design.

### 3 Interaction region

The low photon-photon cross-section, which we have gone to all this effort to achieve, now becomes a problem: the probability for any two photons in the colliding beams to interact will be small. Since photons are neutral, they cannot be focused to another collision point, so they

must have a high probability of interacting in a single collision otherwise the energy used producing them will be wasted. Therefore, to make the interaction probability large, bunches with large numbers of particles will be required. This means many secondary particles can be produced, and can potentially interact with other

<sup>1</sup>For *asymmetric* pairs the two particles will have different phases and the slower particle can even have a phase that is a multiple of  $2\pi$ ; however, the faster particle will always have a phase that is between 0.5 and 1 times the phase for a symmetric pair and so the suppression will still occur.

photons and with each other. It turns out this can lead to serious problems.

The key reactions and cross-sections are ( $h$  here stands for a hadron):

- $\gamma\gamma \rightarrow \text{hadrons}$ ,  $12 \mu\text{b}$  [1].
- $\gamma\gamma \rightarrow e^+e^-e^+e^-$ ,  $6 \mu\text{b}$  [5].
- $\gamma e \rightarrow ee^+e^-$ ,  $150 \text{mb}$  [5].
- $\gamma h \rightarrow \text{hadrons}$ ,  $4 \text{mb}$ .
- $hh \rightarrow \text{hadrons}$ ,  $1200 \text{mb}$ .

For the last two reactions the exact value of the cross-section will depend on the identity of the initial-state hadron(s). Here, to be specific, we have used the  $pp$  cross-section in the last reaction, and used the fact that the ratios  $\frac{\sigma_{\gamma h \rightarrow \text{hadrons}}}{\sigma_{hh \rightarrow \text{hadrons}}}$  and  $\frac{\sigma_{\gamma\gamma \rightarrow \text{hadrons}}}{\sigma_{\gamma h \rightarrow \text{hadrons}}}$  are similar, and essentially measure the hadronic component of the photon (see e.g. [12]), to interpolate the cross-section for  $\gamma h \rightarrow \text{hadrons}$ . We will also need to know how many hadrons are produced per collision in the first and last two processes. This is difficult to estimate, but around  $10^4$  seems a plausible number.

Now let us consider what will happen when two photon bunches collide. Initially the bunches consist only of photons and only the first two reactions are relevant. These will rapidly increase the proportions of electrons, positrons and hadrons: when the number of photons has fallen to  $1 - \epsilon$  of its initial value, the proportion of electrons and positrons will each be  $2\epsilon/3$ , and the proportion of hadrons will be  $10^4\epsilon$ . As the number of hadrons increases, the reaction  $\gamma h \rightarrow \text{hadrons}$  will start to become important; this will happen when the hadron proportion is  $\approx \frac{18}{4000}$ , which is around  $\epsilon \approx 5 \times 10^{-7}$ . Similarly, the reaction  $\gamma e \rightarrow ee^+e^-$  will start to become important when the electron and positron proportions reach  $\approx \frac{18}{300000}$ , which is around  $\epsilon \approx 10^{-4}$ .

This is a disaster! Once these other reactions become important, they will produce yet more electrons, positrons and hadrons, all of which can react with the remaining photons. Effectively, the cross-section for the process  $\gamma\gamma \rightarrow \text{anything}$  is far higher than we expected. This in turn means that the energy requirements for a  $\gamma\gamma$  collider are much higher.

Fortunately this problem can be mitigated by using the fact that these reactions do not take place at a single space-time point but over an

extended region. Firstly, consider the reaction  $\gamma\gamma \rightarrow \text{hadrons}$ . At the parton level, this begins with the reaction  $\gamma\gamma \rightarrow q\bar{q}q\bar{q}$ ; the quarks and antiquarks then radiate further partons, which then hadronise. Due to time dilation, these processes take place over a finite length scale  $\sim \gamma \frac{1}{\Lambda_{QCD}} \sim 0.1 \text{m}$ . Therefore if the colliding photon bunches are shorter than this, they will have passed through each other before hadronisation has taken place, and rather than encountering  $10^4$  hadrons, a counterpropagating photon will only encounter a single quark-antiquark pair.

This mitigates the problem in the hadronic sector, but it still remains significant, and it does nothing about the problem in the electron-positron sector. To go further, we can use a periodic magnetic field to suppress the undesirable reactions, as we did for pair production in the undulator. In this case there is a complication in that particles with different charges are involved: these will follow different trajectories in the magnetic field and hence accumulate different phases after one period of the field. Specifically, a particle of charge  $Z|e|$ , where  $e$  is the electron charge, will be bent through an angle proportional to  $Z$  in a magnetic field; thus the phase

$$\frac{2\pi}{\lambda} \int dl \frac{\theta^2}{2} \quad (8)$$

is proportional to  $Z^2$ . The pair-produced particles have charges  $|Z| = \{1/3, 2/3, 1\}$  for down quarks, up quarks and electrons respectively; thus after one period of the magnetic field, their phases will be in the ratio 1 : 4 : 9.

We have investigated the cancellation that can be achieved in a toy model. We take the length of the magnetic field  $l_B$  equal to the length of the interaction region. The magnetic field has  $N$  periods and is piecewise constant:

$$B = \begin{cases} 0, & x \leq 0 \\ 0, & x \geq l_B \\ c, & 0 < x < l_B \text{ and } |x - \frac{l_B}{N}| < \frac{l_B}{4} \\ -c, & 0 < x < l_B \text{ and } |x - \frac{l_B}{N}| \geq \frac{l_B}{4} \end{cases} \quad (9)$$

For a particle pair produced at  $x$  we calculate the trajectory in the magnetic field from  $x$  to the end of the magnet, and then use equation (8) to calculate the corresponding phase  $\phi_x$ . Finally we integrate over  $x$ , taking the amplitude for production at  $x$  to be proportional to  $e^{-x}$  to take into account the fact that the production

rate will decrease as the bunches pass through each other:

$$\text{Production rate} \propto \left| \int dx e^{-x} e^{2i\phi} \right|^2, \quad (10)$$

where the factor of 2 is due to the fact that the phase for the pair is twice the phase for a single particle.

We show the result for the case  $N = 3$  in Figure 2 as a function of the phase over a single

period of the magnetic field. It is clear that substantial suppression, to a level  $< 10^{-6}$ , is possible. However, as described above the phases for down quarks, up quarks and electrons are different so this suppression cannot be achieved for all three simultaneously. If we wish to minimise the sum of the three suppression rates the best set of phases is  $\{1.81, 7.26, 16.33\} \times \pi$ , where the sum of production rates is suppressed by a factor of 0.00014. For  $l_B = 0.1$  m this corresponds to a magnetic field of 8200 T.

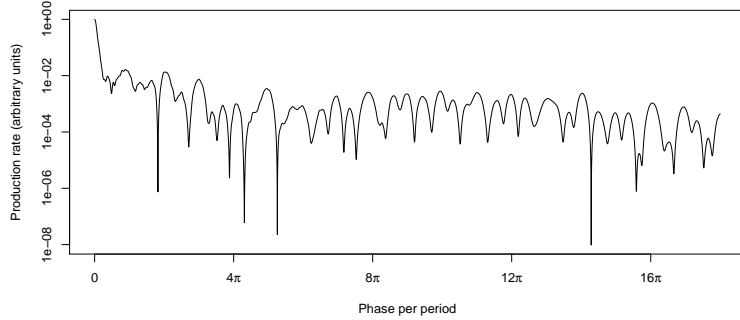


Figure 2: Suppression of pair production for  $N = 3$ .

This is a very strong magnetic field, but it is only required over a very small volume 0.1 m long and perhaps  $10^{-10}$  m wide for less than a nanosecond. Fields of this strength (albeit longitudinal and constant in space rather than transverse and periodic) have already been produced by the Z Pulsed Power Facility over such volumes and timescales [13], so this should be feasible.

A realistic analysis would have to include several complications. Firstly, the above calculation is for production of a single pair, but the first two reactions produce two pairs, so there will be additional suppression. We have also implicitly assumed suppression of each species is equally important, but in fact the production rate for down quarks is lower than for up quarks by a factor of 16. A realistic bunch profile, e.g. a Gaussian, should also be used. Electron-positron pair production has a very long formation length  $\gtrsim 10^5$  m at these energies; thus the reactions involving electrons and positrons could be further suppressed by extending the magnetic field outside the collision region. Another complication is that virtual particles produced in one reaction can be scattered in another reac-

tion, reducing the coherence in equation (10). Fortunately the momentum exchanged in these reactions is very small,  $\mathcal{O}\left(\frac{m^2}{E}\right)$ , so the scattering angles are also small and there will be little loss of coherence. Another point is that we have carried out the above calculations for a fixed momentum  $2.5 \times 10^{14}$  GeV. This is a good approximation for the first four reactions, which produce narrow spectra; however, the last reaction, and its electron equivalent, will produce particles with half this momentum. Finally we have neglected higher order reactions and those which produce heavier particles. These have smaller reaction rates but also much shorter formation lengths which mean they will be suppressed much less.

Some of these complications will increase the suppression and some will decrease it. In the absence of a full calculation we will assume that suppression by about a factor of 1000 is possible. We therefore assume that after suppression the cross section for two photons to produce two pairs of secondary particles,  $\sigma_1$ , is around 10 nb, the cross section for a secondary particle to interact with a photon,  $\sigma_2$ , is around  $100 \mu\text{b}$ , and that if a secondary is likely to interact with  $k$

photons during the bunch crossing there will be an exponentially growing cascade producing  $2^k$  particles.

With these assumptions we can calculate an effective photon-photon cross-section. We proceed by calculating the number of photons per unit area which are just enough so that each photon interacts once. This is given by the equation  $n\sigma_1 2^{n\sigma_2} \approx 1$ , which has the solution  $n \approx 10^{33} \text{ m}^{-2}$ . The effective cross-section is the inverse of this,  $\sigma_{\text{eff}} \approx 10 \mu\text{b}$ . Coincidentally this is close to the value we originally expected neglecting the secondary reactions; it is a factor  $10^5$  lower than the  $pp$  cross-section.

## 4 Detector design

As discussed at the beginning of section 3, because of the low photon-photon cross-section, bunches with large numbers of particles will be required. This in turn means that each bunch collision will release a lot of energy. For a beam width of  $10^{-10} \text{ m}$ , the number of photons per bunch would have to be around  $10^{13}$ , and the energy released per collision would be  $10^{18} \text{ J}$ . Designing the detector to absorb this energy will be a challenge.

At extremely high energies, colliding photons behave very much as hadrons with a reduced cross-section [14]. In particular, this means that most collisions are “soft” – that is, the momentum and energy exchanged are small – and the products emerge predominantly along the beam directions. To be specific, for proton-proton collisions, an approximately flat distribution in *rapidity* is expected. This is almost the same as a flat distribution in *pseudorapidity*,  $\eta = -\ln(\tan \frac{\theta}{2})$ , where  $\theta$  is the angle to the beam direction [15]. I have not been able to find a clear statement in the literature on the expectations for photon-photon collisions, but simulations show similar behaviour [16]. Also, similarly to the case for hadrons, the simulations in [16] show that most events have very low transverse momenta, of the order of 1 GeV, so for large  $\eta$  (small  $\theta$ ),  $\theta \approx \frac{1\text{GeV}}{E}$ , where  $E$  is the particle energy.

The distribution in  $\eta$  is approximately flat up to the maximum kinematically allowed pseudorapidity  $\approx \ln \frac{\sqrt{s}}{\Lambda_{QCD}} \approx 35$ . This implies that the energy is distributed exponentially,  $\frac{dE}{d\eta} \propto e^\eta$ , with most of the energy concentrated in the interval  $34 \lesssim \eta \lesssim 35$ . Since these are soft events,

they will not teach us anything about GUT-scale physics. Therefore one possibility would be to simply allow the collision products at high pseudorapidity to escape into space. However, if they can be absorbed, their energy can be recycled to improve the overall efficiency of the collider. This appears to be feasible for a carefully designed absorber.

When a high energy particle hits a detector, a shower of secondary particles is produced. The absorber must be large enough to contain this shower. The required absorber size depends on the type of particle: at moderate energies, electrons and photons require the smallest detector depths, followed by hadrons, and then muons. Neutrinos interact very weakly and it is impractical to contain them. However, at GUT-scale energies this changes: in ice at  $\sim 10^{14} \text{ GeV}$ , shower depths are around 1 km for electrons and photons, tens of km for muons, but only  $\approx 40 \text{ m}$  for hadrons [17]. Neutrinos interact much more strongly at these energies and a depth of tens of km is also sufficient to absorb them [18].

### 4.1 Hadron absorber

The most challenging part of the detector will be the part which absorbs hadrons, which carry most of the energy and have the shortest absorption length, at very high  $\eta$ , where most of the energy is concentrated. The absorber must not only absorb this energy but also convert it into a useful form capable of being transferred over large distances back to the accelerator. This is particularly challenging for hadronic absorption, since the small absorption depth means the energy density absorbed could be huge. To avoid this, the density of the absorber could be reduced to spread the absorption over a larger volume; however, it would then be difficult to extract heat from the interior of the absorber sufficiently fast. One way to deal with this is to separate the hadron absorber into two components: a small dense target which absorbs the beam energy and radiates it as X-rays, surrounded by a much larger shell which absorbs the X-rays and converts the energy into a useful form.

We begin by considering the latter, and in particular the case where the energy is released as infra-red radiation; this has the advantage that it may be feasible to beam it directly to where it is required without requiring a further conversion step (e.g. a heat engine powering a laser). To minimise the sphere area it should



be constructed of a material with a very high melting point such as graphite. The size of the sphere required seems manageable: for example, for a temperature of  $\approx 4000$  K and a repetition rate of 10 Hz the radius would be  $\approx 300$  km. The thickness of the sphere is given by the requirement that the sphere can absorb the energy of a single bunch collision without melting. For graphite, the energy absorbed per atom to raise the temperature to  $\approx 4000$  K is around 0.3 eV. The energy emitted in each direction near  $\eta = \pm 35$  is  $0.5 \times 10^{13} \times 10^{15}$  GeV; thus at least  $1.5 \times 10^{37}$  atoms of carbon will be required. This corresponds to a mass  $m \approx 3 \times 10^{11}$  kg and a thickness of only around  $100 \mu\text{m}$ .

The repetition rate determines the total number of absorbers required: if the full solar luminosity is used, with 100% efficiency (see also section 6.1), there would be  $\approx 2 \times 10^7$  interaction points, and so  $\approx 4 \times 10^7$  absorbers, requiring a total carbon mass of around  $10^{19}$  kg. This is relatively modest: it is about 10% of the total carbon in the atmosphere of Venus.

The thickness of the sphere determines the X-ray energy; this must be such that the absorption depth for the X-rays is similar to the thickness of the graphite shell. A smaller absorption depth would result in most of the X-rays being absorbed near the inner surface of the shell; the temperature in this part of the shell would then rise above the melting or sublimation point. Conversely, a larger absorption depth would mean that most of the X-rays would penetrate the shell without being absorbed. For the particular case of graphite, a thickness of  $100 \mu\text{m}$  corresponds to an X-ray energy of 4 keV [19, 20].

We now turn to the dense target. This should be a cylinder about 40 interaction lengths in depth, and one interaction length in radius [21]. The nuclear interaction length depends on the density,  $\lambda = k/\rho$ , where  $k \approx 100 \text{ g cm}^{-2}$  for most materials. Thus the number of atoms in the target scales as  $\rho^{-2}$ . Therefore at low temperatures, where most of the energy ends up in ions and electrons, the temperature goes as  $\rho^2$  and the energy density in ions and electrons goes as  $\rho T \propto T^{3/2}$ . The energy density in photons increases faster; it goes as  $T^4$ . There is thus a crossover temperature, which is hundreds of eV for most light elements, above which most of the energy ends up in photons. In this regime the energy density in photons continues to increase as  $T^4$  while the energy density

in ions and electrons increases as  $T^{7/3}$ .

This means that if we choose the target parameters so that the temperature after the hadrons are absorbed is above the crossover temperature, most of the energy will indeed end up in X-ray photons. To obtain photons with the desired X-ray temperature (4 keV in the example above), the volume of the absorber should be chosen so that the energy density of black-body radiation at that temperature, multiplied by the absorber volume, is equal to the absorbed energy. This gives a volume of about  $250 \text{ m}^3$  for the example above. This in turn determines the interaction length and hence the density and mass of the absorber. The absorber turns out to be rather small, with a mass of  $\sim 10^5$  kg, a radius of a metre or so, and a length of tens of metres.

## 4.2 Other components

Further absorbers will be needed behind the hadronic absorber for electromagnetically interacting particles, muons and neutrinos. These should be easier to design than the hadronic absorber, since the majority of the energy is in hadrons.

At lower  $\eta$ , the energy density is exponentially lower. Therefore the detector design becomes much less challenging and the detector can be designed with particle identification and tracking in mind, rather than energy recovery. The smaller energy density means this part of the detector can be much more compact.

## 5 Accelerator design

Given the above, the requirements for the muon accelerator are:

- A muon energy around  $6 \times 10^{14}$  GeV.
- A bunch interval of 0.1 s.
- $10^{13}$  muons per bunch.

A circular accelerator with this energy would have huge synchrotron radiation losses unless its radius was huge,  $\gtrsim 10^{30}$  m (larger than the Hubble radius). A linear accelerator can be much shorter than this so would definitely be preferred. With current conventional accelerator technology, gradients of 100 MV/m are possible. It seems plausible that this could be increased

to, say, 1 GV/m by the time the collider proposed here is built, which would give a length of  $6 \times 10^{14}$  m (about a light month).

It is possible advanced accelerator technology could reach much higher gradients than this. Possibilities include acceleration in dielectric structures, plasma wakefield accelerators, or acceleration in crystal channels (see [22] for a review). These might achieve gradients of 1 TV/m, which would reduce the required length to less than  $10^{12}$  m – roughly the radius of Jupiter’s orbit.

Note that the total length of the collider would be slightly over double the length calculated above, since two muon accelerators are required, one in each direction. An additional allowance must be made for the undulators and for the detectors, but these will be orders of magnitude smaller.

The accelerator must also be designed to avoid excessive losses due to absorption of virtual photons [23]. These can be estimated using the Weizsäcker-Williams approximation. In the accelerator frame, the Coulomb field of the muon becomes a short electromagnetic pulse. The energy of this pulse in an annulus between radii  $R$  and  $2R$  from the beam is  $\gamma\alpha/2R$ , and the photon energy is  $\omega \approx \gamma/R$ . For, say,  $R = 0.1$  m we find the pulse has an energy of about 50 MeV and  $\omega \approx 10$  GeV. Absorption lengths at 10 GeV are tens of  $\text{g cm}^{-2}$  (with high- $Z$  materials having the shortest absorption length), and a density of  $\sim 1 \text{ g cm}^{-3}$  seems reasonable to allow for gaps between components etc., which would give an absorption length of tens of centimetres. This would lead to losses of around 100 MeV/m, safely lower than the likely acceleration gradient. However, large densities of high- $Z$  materials very near ( $\lesssim 1$  cm) to the beam should be avoided. (For electrons this problem would be orders of magnitude worse; this is another reason to prefer muons to electrons as the primary particles.)

Note that ref. [23] assumes that the beampipe must be solid and very thick to contain the pressure of the electromagnetic fields which accelerate the high energy particles, and so concludes that the inner radius of the beampipe must be very large to prevent large losses by absorption of virtual photons. However, this is too pessimistic: it assumes the pressure is static and must be balanced by static forces in the pipe, whereas in fact it will only be

present while the very short bunches are passing, so it can be balanced by inertial forces (the pipe can ‘stretch’ while the bunch passes). In a plasma wakefield accelerator there is no beampipe anyway.

We have so far discussed the accelerator as if there is only one. However, this is not the case; there will need to be a large number of accelerators running in parallel. If there is one pair per interaction point there will be about  $4 \times 10^7$  of them.

Note that the bunch frequency is relatively small by current accelerator standards. It may be better to increase this parameter and reduce the number of accelerators, and then separate the bunches before the undulators. For example, the ILC design has 6560 bunches per second (admittedly smaller bunches than proposed here: there will be  $2 \times 10^{10}$  electrons / positrons per bunch) [1] – using this bunch frequency, approximately 60000 accelerators would be required. This would have the advantage of greatly reducing the mass required to construct the accelerators. It is hard to know how much mass would be required per unit length, since this depends so much on the acceleration technology. Assuming, say,  $100 \text{ kg m}^{-1}$ , and 60000 accelerators each of length  $6 \times 10^{14}$  m, the total mass required would be  $4 \times 10^{21}$  kg. For comparison, the mass of Mercury is  $3 \times 10^{23}$  kg, so this seems manageable. Without increasing the bunch frequency, the mass required would be  $2 \times 10^{24}$  kg. It is likely that much of this would be heavy elements, the main accessible sources of which in the solar system are the terrestrial planets, which have a total mass of  $1.2 \times 10^{25}$  kg. This might well be insufficient, depending on the particular elements required.

One final remark about the length: the most important consideration in the design of the collider is the energy efficiency. It may be therefore that a lower acceleration gradient, and hence longer accelerators, will be preferred if it is more efficient.

## 5.1 Location

It would make sense for the interaction points to lie as near the solar system as possible, since this will decrease the total distance construction materials will have to be transported. However, the accelerators and detectors will need to lie almost on a straight line to reduce synchrotron losses. The problem with this is that an initially lin-

ear arrangement will not remain straight if the components are orbiting (except for the courageous option of the components moving directly towards/away from the Sun and hence aiming one of the beams directly at the inner solar system). One solution is to give the whole collider an initial transverse velocity away from the Sun. The outer parts would continue moving at approximately constant velocity, while the central parts would be slowed by the Sun's gravity, and would need to be accelerated to overcome this. The total  $\Delta V$  required for this over the 400-year lifetime of the collider is approximately  $\frac{75000}{d^2} \text{ km s}^{-1}$ , where  $d$  is the distance from the Sun in astronomical units. This is less than  $100 \text{ km s}^{-1}$  for distances around Neptune's orbit or greater.

Much larger  $\Delta V$ s would probably be required to construct the accelerator in a reasonable timescale. For the length of  $6 \times 10^{14} \text{ m}$  considered above, velocities in excess of  $600 \text{ km s}^{-1}$  would be needed for the outermost accelerator components in order to get them to the required location within 30 years. However, these velocities are only required for the ends of the accelerators; smaller velocities are enough for the inner parts and the detectors.

Are these speeds energetically feasible? Suppose a total mass of  $4 \times 10^{21} \text{ kg}$  must be accelerated to an average speed of  $500 \text{ km s}^{-1}$ ; the total kinetic energy required is then  $10^{33} \text{ J}$ . This is "only" 0.1 years of the total solar luminosity, and hence very small compared to the energy which will be required to run the accelerator. So this is OK.

## 6 Energy transfer

The energy required by the collider can be collected by a Dyson swarm [24, 25] – a large number of solar power collectors that completely enclose the Sun. The energy required to achieve the desired integrated luminosity is  $5 \times 10^{36} \text{ J}$ ; a Dyson swarm could collect this much energy in 400 years.

In addition, time would be required to construct the Dyson swarm. The speed of construction can increase exponentially with time, as the energy from the partially completed swarm is used to manufacture further collectors. One estimate is that constructing a complete swarm would take 30 years and that the mass required would be less than the mass of Mercury [26].

The energy collected by the Dyson swarm must then be transmitted to the accelerator, up to  $6 \times 10^{14} \text{ m}$  away. Two methods proposed for transmitting power over long distances in space are microwaves (transmitted by antennae and received by rectennas) and light waves (transmitted by lasers and received either by photovoltaics or by some sort of heat engine). The minimum size for the transmitters and receivers is given by the diffraction limit. For electromagnetic waves of wavelength  $\lambda$ , the transmitter and receiver sizes  $s_r$  and  $s_t$  are related to the distance  $L$  by  $s_r s_t \gtrsim L\lambda$ . Even for the largest distances considered, the required transmitter and receiver sizes are tens of km for lasers and up to thousands of km for microwaves.

However, a diffraction-limited receiver would be exposed to huge power densities. The power being transmitted is the total solar luminosity; for a receiver tens (thousands) of km across, the power density is  $\sim 10^{17} (10^{13}) \text{ W m}^{-2}$ . This seems far too high, so either the receiver should be larger than the diffraction limit, or there should be many of them. Whether laser or microwave transmission should be used will depend on which has better energy efficiency, and to some extent also on the mass required. The mass required will be at most similar to the mass required for the Dyson swarm, since the total power absorbed is the same.

A possible approach which might help efficiency would be to use the same wavelength for both power transmission and particle acceleration. Conventional accelerator technology uses microwaves, and plasma acceleration uses lasers. Therefore it would in principle be possible to redirect the incoming electromagnetic radiation directly into the accelerator, which would avoid two conversion steps: microwave (laser) to electrical, and electrical to microwave (laser).

### 6.1 Energy efficiency

It is very difficult to estimate the energy efficiency of possible future technologies without knowing the details. Optimistically, however, and bearing in mind that the Carnot efficiency for recovering waste heat could be very high since temperatures in the outer solar system are very low, we might assume something like the following:

- $a = 80\%$  of the solar luminosity is delivered to the accelerator.

- The accelerator efficiency is  $b = 60\%$ .
- $c = 90\%$  of the collision energy is recovered and delivered back to the accelerator.

The total efficiency with these assumptions is  $\frac{ba}{1-bc} = 104\%$ . Thus an efficiency around 100% appears possible.

## 7 Conclusions

We have shown that a feasible design for a GUT-scale collider is a photon collider, with the photons produced as undulator radiation from muon beams of slightly higher energy. Such a collider would be approximately  $10^{15}$  m long, and would require 400 years to collect  $\mathcal{O}(10)$  interesting events if powered by the full solar luminosity. In order to control the energy requirements, secondary reactions in the collision region must be suppressed; we have shown that this can be achieved if the collisions take place in a strong periodic magnetic field.

There do not appear to be any fundamental reasons why a collider of the type described in this paper could not be built. However, there is the issue of vacuum stability to consider. We are assuming no physics beyond the Standard Model; however, the Standard Model vacuum is probably metastable [27], raising the possibility that GUT-scale collisions could cause vacuum decay. Even if it can be shown that this would not occur within the Standard Model, there is the possibility of new physics changing this conclusion.

No cosmic rays have been observed with energies near the GUT scale. However, a greater understanding of the sources of cosmic rays might show that cosmic rays with these energies do exist and that they sometimes collide within or near these sources; if enough collisions have occurred in our past lightcone this would show that vacuum stability is not a problem. Similarly, greater understanding of cosmology may show that the early universe reached a temperature above  $T_{\text{GUT}}$ , which would allow a similar conclusion to be reached. This issue should be resolved before construction begins.

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